Grade 6 Math Circles

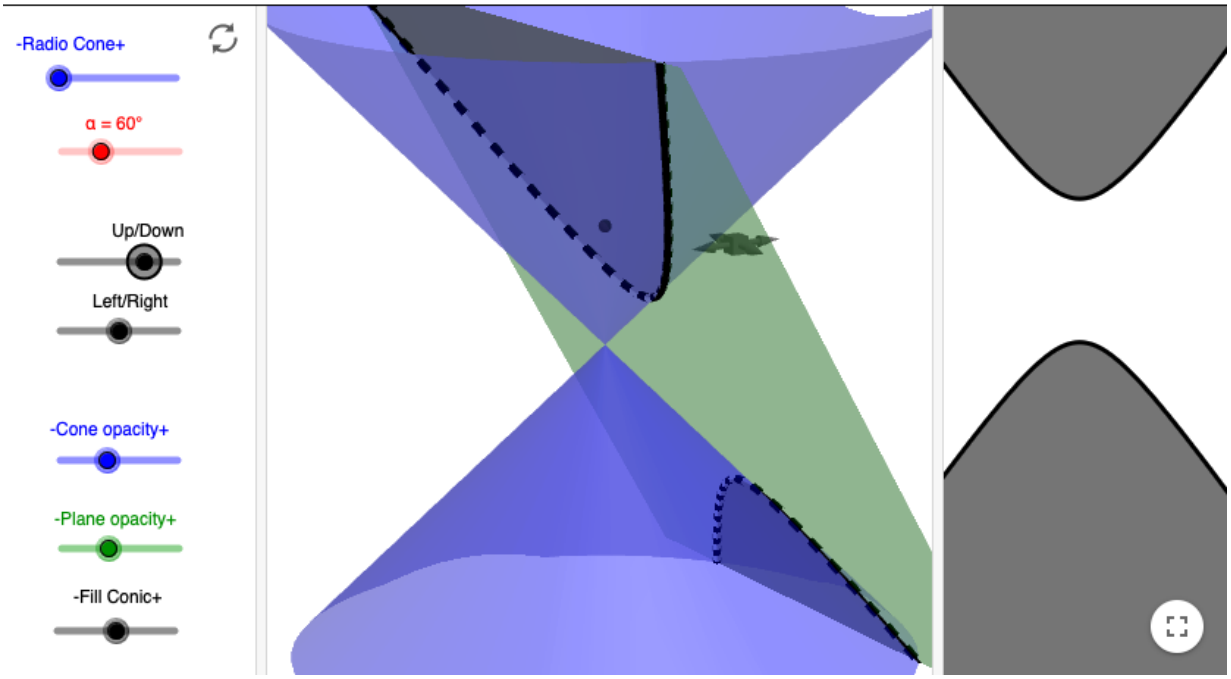
October 10/11/12/16

Exploring Conics Problem Set

Visit this interactive website to help you explore the various types of conic sections:
https://www.geogebra.org/m/JCAgPuzC#material/pCg8NFVT

Right away, you will notice that there are two cones being used instead of one. This is because, unlike the conic sections we explored, real conics might have two parts, as in the case of the hyperbola.

Using the slider for angle (the red slider for a) and the slider for and up/down (one of the black slider), try moving the green surface around to make different slices. The right side of the screen shows you what your slice is. For example, if we make $a = 60°$ and shift up a little, we get the following:

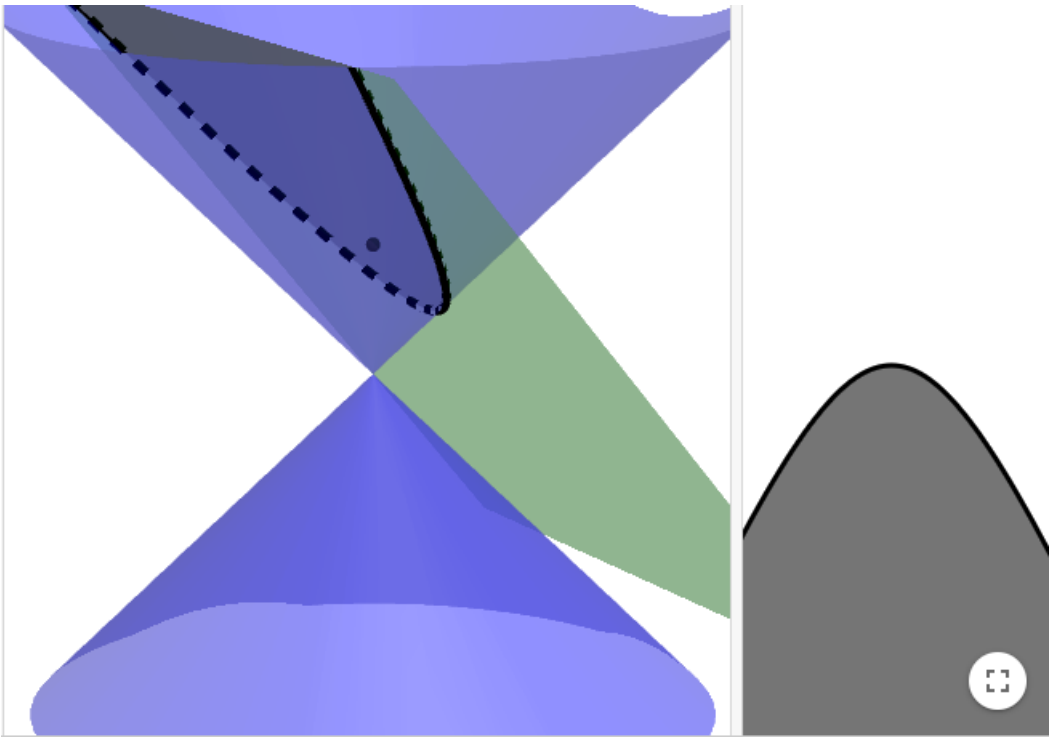


This is a real hyperbola - a real hyperbola always has two parts like this. By using only the two sliders mentioned, try to answer the following questions.

1. (a) Find a value of a and move the Up/Down slider to create a different hyperbola.
   (b) Find a value of a and move the Up/Down slider to create an ellipse.
   (c) Find a value of a and move the Up/Down slider to create a circle.
   (d) Find a value of a and move the Up/Down slider to create a two lines (They will look like an X).

2. When playing around with the sliders, did you come across a shape on the right that you haven't seen before?
   Hint: Remember that a real hyperbola has two parts.

3. In question 2, you might have noticed that we can create what looks like a hyperbola, but only has one piece. This final conic section is called a *parabola*. It looks very similar to the fake hyperbola that we drew, but is actually a different shape! Here's an example of a parabola:



   Watch this YouTube video to learn how to draw a parabola:
   https://www.youtube.com/watch?v=7zJPL0b_XxI The video presents two different ways to draw the parabola. Use whichever seems easier.

   We said that the Six Point Property is true for any conic section. Verify that it works for the parabola that you drew.

4. For our final piece of exploration, go to this interactive website to explore Pascal's theorem:
   https://www.geogebra.org/m/UH96KVSY

   Here, you can move around the six points by dragging them to see that the Six Point Property holds for any conic section.

   We can change the values of the parameters as follows to get the different conic sections.

   (i) A circle: $a = 1$, $b = 1$, $c = 0$, $d = 0$, $e = 12$
   (ii) An ellipse: $a = 1$, $b = 2$, $c = 0$, $d = 0$, $e = 12$
   (iii) A hyperbola: $a = 1$, $b = -1$, $c = 0$, $d = 0$, $e = 12$
   (iv) A parabola: $a = -1$, $b = 0$, $c = 0$, $d = 1$, $e = 0$